Yuan's Gap Principle for generalized simultaneous Pell equations

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We consider the following system of equations

$$|a_1x^2 - a_2z^2| = 4; \quad |b_1y^2 - b_2z^2| = 4$$

in rational integers x, y and z, where a_1, a_2, b_1 and b_2 are given positive integers. We discussed a geometric gap principle. It has its nature in geometry in archimedean plane. It is an exponential gap priciple. This means it is considerably strong. However, it is a three term gap principle which estimates the third solution (from below) in terms of the first solution. Unfortunately, the last point restricts its power so that we could only estimate the number of positive solution by 3.

On the other hand, Yuan found a gap principle for a smaller class of systems (subject to $a_1 = b_1 = 4, a_2, b_2 \in 4\mathbf{Z}$). It has its nature in geometry in p-adic plane. It is also an exponential gap principle. More importantly, it estimates the third solution (from below) in terms of the second solution.

Therefore, it opens a way of estimating the number of positive solutions by 2. By combining the gap principles, this bound will be achieved except for finite numbers of explicit combinations of parameters.

However, in our general case Yuan's gap principle needs to be combined with more detailed information on the first two solutions. Unfortunately, there are infinite class of combinations of parameters for which Yuan's gap principle has no effect or very weak effect.

In this talk, we will analyse the strength of Yuan's gap principle.